Inductive Reasoning and Conjecture

Making Conjectures  Inductive reasoning is reasoning that uses information from different examples to form a conclusion or statement called a conjecture.

Example 1  Write a conjecture about the next number in the sequence 1, 3, 9, 27, 81.

Look for a pattern:
Each number is a power of 3.

<table>
<thead>
<tr>
<th>1</th>
<th>3</th>
<th>9</th>
<th>27</th>
<th>81</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^0$</td>
<td>$3^1$</td>
<td>$3^2$</td>
<td>$3^3$</td>
<td>$3^4$</td>
</tr>
</tbody>
</table>

Conjecture: The next number will be $3^5$ or 243.

Example 2  Write a conjecture about the number of small squares in the next figure.

Look for a pattern: The sides of the squares have measures 1, 2, and 3 units.

Conjecture: For the next figure, the side of the square will be 4 units, so the figure will have 16 small squares.

Exercises

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. $-5, 10, -20, 40$  Pattern: Each number is $-2$ times the previous number.  Conjecture: The next number is $-80$.

2. $1, 10, 100, 1000$  Pattern: Each number is 10 times the previous number.  Conjecture: The next number is 10,000.

3. $1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}$  Pattern: Each number is $\frac{1}{5}$ more than the previous number.  Conjecture: The next number is $\frac{9}{5}$.

Write a conjecture about each value or geometric relationship. 4–7. Sample answers are given.

4. $A(-1, -1), B(2, 2), C(4, 4)$

Points $A$, $B$, and $C$ are collinear.

5. $\angle 1$ and $\angle 2$ form a right angle.  $\angle 1$ and $\angle 2$ are complementary.

6. $\triangle ABC$ and $\triangle DBE$ are vertical angles.  $\angle ABC$ and $\angle DBE$ are congruent.

7. $\triangle E$ and $\triangle F$ are right angles.  $\angle E$ and $\angle F$ are congruent.
Find Counterexamples  A conjecture is false if there is even one situation in which the conjecture is not true. The false example is called a counterexample.

Example  Find a counterexample to show the conjecture is false.

If $\overline{AB} \cong \overline{BC}$, then $B$ is the midpoint of $\overline{AC}$.

Is it possible to draw a diagram with $\overline{AB} \cong \overline{BC}$ such that $B$ is not the midpoint? This diagram is a counterexample because point $B$ is not on $\overline{AC}$. The conjecture is false.

Exercises

Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

1. If points $A$, $B$, and $C$ are collinear, then $AB + BC = AC$.
   False; $C$ could be between $A$ and $B$.

2. If $\angle R$ and $\angle S$ are supplementary, and $\angle R$ and $\angle T$ are supplementary, then $\angle T$ and $\angle S$ are congruent.
   True

3. If $\angle ABC$ and $\angle DEF$ are supplementary, then $\angle ABC$ and $\angle DEF$ form a linear pair.
   False; the angles could be nonadjacent.

4. If $\overrightarrow{DE} \perp \overrightarrow{EF}$, then $\angle DEF$ is a right angle.
   True