Congruent Triangles

Congruence and Corresponding Parts
Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, \( \triangle ABC \cong \triangle RST \).

<table>
<thead>
<tr>
<th>Third Angles Theorem</th>
<th>If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.</th>
</tr>
</thead>
</table>

**Example** If \( \triangle XYZ \cong \triangle RST \), name the pairs of congruent angles and congruent sides.

\[
\angle X \cong \angle R, \quad \angle Y \cong \angle S, \quad \angle Z \cong \angle T \\
XY \cong RS, \quad XZ \cong RT, \quad YZ \cong ST
\]

**Exercises**
Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1. \( \triangle ABC \cong \triangle JKL \)
\[
\angle A \cong \angle J; \quad \angle B \cong \angle K; \\
\angle C \cong \angle L; \quad AB \cong JK; \\
BC \cong KL; \quad AC \cong JL \\
\triangle ABC \cong \triangle JKL
\]

2. \( \triangle ABC \cong \triangle DCB \)
\[
\angle A \cong \angle D; \quad \angle ABC \cong \angle DCB \\
\angle ACB \cong \angle DBC; \quad AC \cong BD \\
AB \cong DC \quad \triangle ABC \cong \triangle DCB
\]

3. \( \triangle JKM \cong \triangle LMK \)
\[
\angle J \cong \angle L; \quad \angle JKM \cong \angle LMK; \\
\angle K \cong \angle M; \quad JK \cong ML \\
\triangle JKM \cong \triangle LMK
\]

4. \( \triangle FGE \cong \triangle KLJ \)
\[
\angle E \cong \angle J; \quad \angle F \cong \angle K; \\
\angle G \cong \angle L; \quad EF \cong JK; \\
EG \cong JL; \quad FG \cong KL \\
\triangle FGE \cong \triangle KLJ
\]

5. \( \triangle RST \cong \triangle TSU \)
\[
\angle R \cong \angle T; \quad \angle RSU \cong \angle TSU; \\
\angle RST \cong \angle TSU \\
\triangle RST \cong \triangle TSU
\]

6. \( \angle C \cong \angle B; \quad \angle ABC \cong \angle DCB; \\
\overline{AC} \cong \overline{DB}; \quad \overline{BC} \cong \overline{CB} \\
\triangle ABC \cong \triangle DCB
\]

Suppose \( \triangle ABC \cong \triangle DEF \)

7. Find the value of \( x \). 27.8
8. Find the value of \( y \). 35

Chapter 4 19
Glencoe Geometry
4-3 Study Guide and Intervention (continued)

Congruent Triangles

Prove Triangles Congruent Two triangles are congruent if and only if their corresponding parts are congruent. Corresponding parts include corresponding angles and corresponding sides. The phrase “if and only if” means that both the conditional and its converse are true. For triangles, we say, “Corresponding parts of congruent triangles are congruent,” or CPCTC.

Example Write a two-column proof.
Given: $AB \cong CB, AD \cong CD, \angle BAD \cong \angle BCD$  
$BD$ bisects $\triangle ABC$.

Prove: $\triangle ABD \cong \triangle CBD$

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1. AB \cong CB, AD \cong CD$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$2. BD \cong BD$</td>
<td>2. Reflexive Property of congruence</td>
</tr>
<tr>
<td>$3. \angle BAD \cong \angle BCD$</td>
<td>3. Given</td>
</tr>
<tr>
<td>$4. \angle ABD \cong \angle CBD$</td>
<td>4. Definition of angle bisector</td>
</tr>
<tr>
<td>$5. \angle BDA \cong \angle BDC$</td>
<td>5. Third Angles Theorem</td>
</tr>
<tr>
<td>$6. \triangle ABD \cong \triangle CBD$</td>
<td>6. CPCTC</td>
</tr>
</tbody>
</table>

Exercises

Write a two-column proof.

1. Given: $\angle A \cong \angle C, \angle D \cong \angle B, AD \cong CB, AE \cong CE$,  
$AC$ bisects $BD$.

Prove: $\triangle AED \cong \triangle CEB$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1. \angle A \cong \angle C, \angle D \cong \angle B$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$2. \angle AED \cong \angle CEB$</td>
<td>2. Vertical angles are $\cong$.</td>
</tr>
<tr>
<td>$3. AD \cong CB, AE \cong CE$</td>
<td>3. Given</td>
</tr>
<tr>
<td>$4. DE \cong BE$</td>
<td>4. Definition of segment bisector</td>
</tr>
<tr>
<td>$5. \triangle AED \cong \triangle CEB$</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>

Write a paragraph proof.

2. Given: $BD$ bisects $\angle ABC$ and $\angle ADC$,  
$AB \cong CB, AB \cong AD, CB \cong DC$

Prove: $\triangle ABD \cong \triangle CBD$

We are given $BD$ bisects $\angle ABC$ and $\angle ADC$. Therefore $\angle ABD \cong \angle CBD$ and $\angle ADB \cong \angle CDB$ by the definition of angle bisectors. By the Third Angle Theorem, we find that $\angle A \cong \angle C$. We are given that $AB \cong CB, AB \cong AD$, and $CB \cong DC$. Using the substitution property, we can determine that $\triangle ABD \cong \triangle CBD$. Finally, $BD \cong BD$ using the Reflexive Property of congruence. Therefore $\triangle ABD \cong \triangle CBD$ by CPCTC.